# Temperaments

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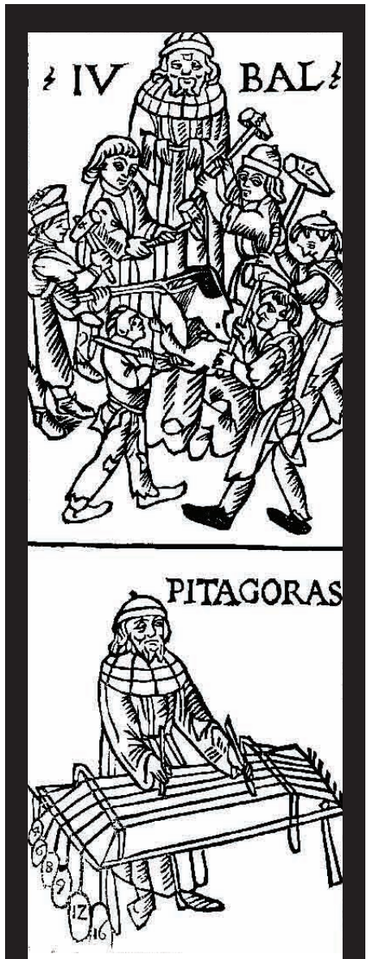
This is the first in a series which aims to explain the history and structure of some of the various temperaments used in period performance. This is a subject which often becomes very quickly complicated and mathematical. In each issue, Edward Dean introduces a different tuning system, and in a language which we hope makes this topic approachable for all.

Part One: Pythagorean

Walking past a workshop of blacksmiths, Greek mathematician and philosopher Pythagoras (570-495 BC) was struck by the harmonious sound created when various hammers of different sizes struck anvils at the same time; he later returned to the workshop to explore the possibility of a relationship between hammer size and pitch. [Fig. 1]

Although this is reckoned by many to be a myth (as this sort of pattern only occurs in relation to string length rather than hammer size),




[Fig. 1]

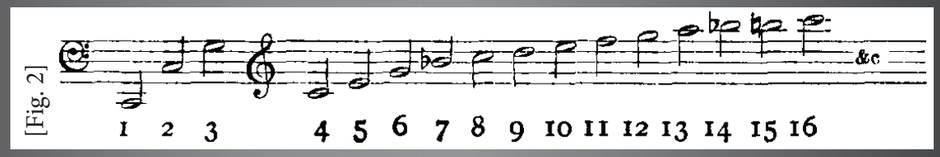
it serves to illustrate the popularly shared belief that Pythagoras was responsible for the discovery of the 'harmonic series' [Fig. 2].

Reckoning the pure fifth to be one of the most consonant sounds, Pythagoras set about creating a tuning system based upon this interval alone. However, when stacking pure fifth upon pure fifth, we realise that when the twelfth fifth is reached, although we have travelled seven octaves in theory (or 84 semitones), we reach a note that is markedly sharper than that achieved by tuning seven pure octaves from the initial note: this can be seen by the 'Pythagorean spiral' diagram. [Fig. 3]

To achieve this 'closed' cycle rather than a spiral, Pythagoras sacrificed one of these pure fifths (supposedly g# and e*b*) and placed the whole of the 'impurity' or 'comma' on it; this interval is what we refer to as the 'wolf', which, given its jarring nature, is to be avoided.

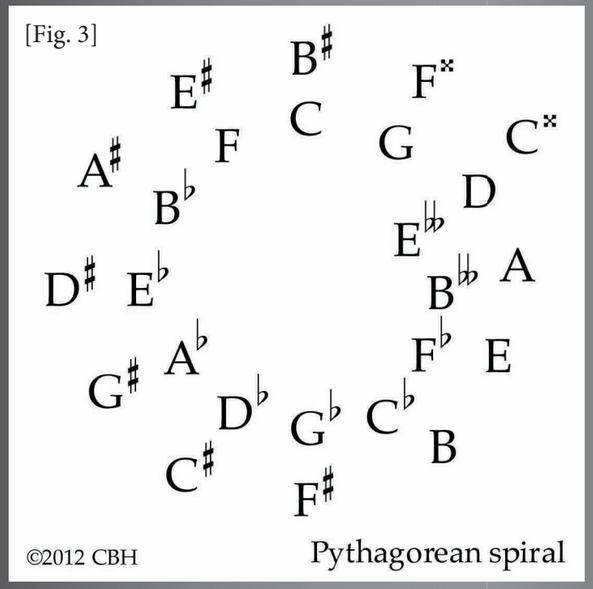
As pure fifths are 701.96 cents wide, by doing this, Pythagoras had narrowed this fifth by the comma of 23.46 cents (which is almost equivalent to a quarter of a semitone), which gives the 'Wolf' fifth a width of around 678.5 cents.






[Fig. 2] The harmonic series.
Pythagoras realised that for whatever pitch a string sounded at when at its original length (C, for instance), the same pitch an octave higher (c) could be created by a string of exactly half its length. He also found that if the same original string had a tangent placed two thirds of the way down its length, a pure, beatless fifth an octave higher (g) could be 'heard'.

[Fig. 3]




[Fig. 3]. The difference between these two notes is around 23.46 cents (referring back to Issue 1, we recall that 1 cent = 1/1200th of an octave, or 1/100th of an equally-tempered semitone), and this value we now refer to as the 'Pythagorean comma'. His aim was to make a system where not only the octaves 'lined up' but also one that could still be tuned using a cycle of fifths, which he succeeded with what we now think to be earliest western tuning system in: 'Pythagorean'.

Whereas later tuning systems altered or, in other words, 'tempered' intervals, Pythagoras's pure fifths and resulting 'Wolf' cause many aficionados to call 'Pythagorean' more of a tuning system than a true 'temperament'. 'Pythagorean' is said to be a pure or 'just' system/temperament as any relationship between two notes, just like the harmonic series, can be expressed as a ratio of a small 'integer' or whole number.

Following Boethius' lead in the 4th century, the Notre Dame school in the 13th and 14th centuries reckoned that only pure fifth-based tuning systems were acceptable - perhaps stemming from the perfect fifth's very 'Trinitarian' frequency ratio of 3:2.

Pythagorean tuning also offers three pure major thirds (indicated by the straight lines on figure 4); although this too is

'Trinitarian', it is unlikely that they were the temperament's central features as their positions make them pretty inaccessible to the modes commonly used at the time. Apart from these pure thirds, 'Pythagorean' has relatively wide major thirds and sixths, and narrow minor thirds and sixths.

Given the importance of pure fifths in the music of the pre-Renaissance, one might think that this music would